Too Hot, Too Cold or Just Right?

Temperature control is important for human comfort, and a significant portion of a family's energy budget is often devoted to their home's heating and air conditioning systems. In many industrial and food processing applications, temperature control is even more critical. Whether the product is steel, pharmaceuticals, plastics, automobile windshields, electronics or ice cream, temperatures that are too high or too low can be disastrous. The wrong temperature during production, transportation or storage can make a product unusable—and sometimes the wrong temperature can lead to fatal accidents or illnesses.

Temperature Controls in Dairy Production
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While linear gauges are adequate for most temperature measurements, non-linear gauges offer advantages when the temperature must be precisely controlled near a specific target. Consider, for example, an industrial operator who needs to know at a glance the temperature of a liquid which can vary from 0.01°C to 100°C. The operator only needs a rough estimate of higher temperatures, but needs very precise information when the temperature is near the freezing point, 0°C.

Task: You are to design a mathematical function which translates temperatures from a sensor into an easy-to-read visual indicator. The function must position the gauge appropriately on a scale provided.

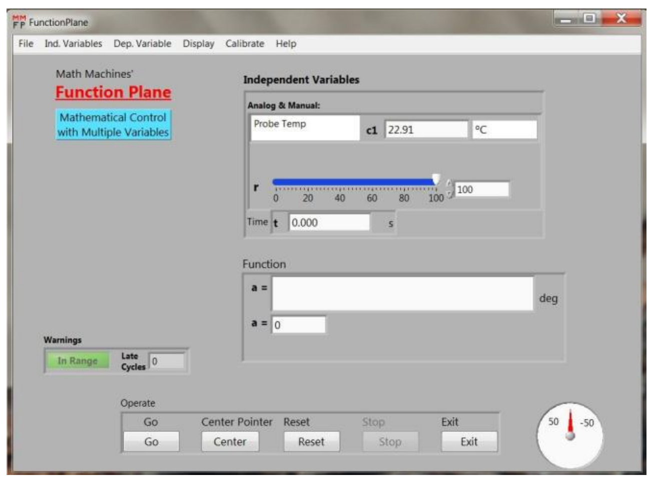
Additional Materials: Temperature Probe, cup, water, ice, Modified Celsius Gauge, Protractor overlay, scientific or graphing calculator.

Math Machines Program:

Function Plane

Activity File: RightTemp

Connect the SensorDAQ and insert the temperature probe in Ch. 1. Start the Function Plane software and answer "yes" to the question about analog probes.[1]



[1] To test the activity without a SensorDAQ and temperature probe, you much use the activity file, "RightTemp-alt".

1. The desired function must calculate a gauge position (in angle degrees) for any input temperature (in degrees Celsius). Compare the Modified Celsius gauge which the customer provided with the transparent protractor overlay and identify 5 ordered pairs for the function. Enter the values for these 5 ordered pairs in the table below.

Independent Variable Temperature (°C)	Dependent Variable Gauge Position (angle °)

2. Plot the 5 ordered pairs on a graph using paper and pencil, a spreadsheet, a graphing calculator, or another technology. Use Temperature as the independent (horizontal) variable and Gauge Position as the dependent (vertical) variable.
   a. Is the relationship linear? ____________
   b. Describe the relationship in words below. Also attach your graph.[2]

3. Consider the non-linear operations which are available on your scientific or graphing calculator, including "√(x)", "x^{-1}," "x^2," "LOG," "LN," "SIN," "COS," "TAN," and others. Experiment if necessary to identify one or more of these operations which you could use as an intermediate step on the 5 Celsius temperatures to yield quantities which do have a linear relationship with the desired gauge positions. If there is more than one valid operation, select the simplest one. In the table below, copy the first and last columns from the table above and complete the center column with the operation you select and the 5 intermediate values.

Independent Variable Temperature (°C)	Intermediate Operation ____________________	Dependent Variable Gauge Position (angle °)

[2] If you use a graphing calculator or cannot attach the graph for some other reason, draw a sketch instead.

4. Explain why you selected the operation you chose.

5. Determine the precise linear function (in the form “y = mx + b”) which will always convert your intermediate value (column 2) to the gauge position (column 3). If necessary, plot a graph showing the gauge position as a function of the intermediate values to help you determine the slope, “m,” and the y intercept, “b,” for this linear function.

Slope, m = ______________

Y Intercept, b = __________

Gauge position (angle degrees) = ________ x intermediate value + __________

6. Substitute the operation you selected in step 3 for the “intermediate value” in the function above to determine a single function which will calculate Gauge Position (in angle degrees) from the Temperature (°C). Enter this function below.

7. Check your function against the “Partial List of Available Functions” below. Each of these functions must be in lower case and must be followed by a quantity in parentheses. The quantity in parentheses can be either a number, a variable (c1, r or t) or an expression using any of these variables. Any multiplications must be entered explicitly using the “*” symbol. If necessary, change the syntax to match the software’s requirements. For testing purposes, use “r” as the independent variable. You will change this later to “c1” for actual temperature measurements. Write your modified function below at right and type it into the software’s Function Box.

<u>Partial List of Available Functions:</u>

r^2	r squared
sqrt(r)	Square root of r
(r)^-1	Inverse of r
log(r)	Base 10 logarithm of r
ln(r)	Natural logarithm of r
sin(r)	Sine of r (r in radians)
cos(r)	Cosine of r (r in radians)
tan(r)	Tangent of r (r in radians)

<u>Sample Combinations:</u>

15*sqrt(r)
log(r/40)
50+50*cos(r^2)

8. Click “Go” to run the program and test your function for different values of r. You can change the value of r either by moving the “slider” or by typing in the value you want. Verify in particular that your function reproduces the ordered pairs you identified in step 1.

9. Identify the practical domain for this use of your function:

   a. What happens if you set the value of “r” to zero?

   b. What is the lowest value for “r” which keeps the temperature gauge in its marked region?

   c. The domain is ______________________________

10. What is the practical range for this use of your function? (That is, what range of angle degrees can be displayed on the gauge?)

11. Replace “r” with “c1” in your function so the independent variable will be the actual temperature measurement from the temperature probe. Experiment with ice water to find the lowest temperature you can actually display on your gauge. Describe the method you used and the result below.