

# Magnetic Fields and Moving Charged Particles    Vector Cross Products

## Uses of Magnetic Fields

Magnetic fields are often used to steer electrically charged particles. Research examples include the use of magnetic fields in cyclotrons, but the same process is used in medicine (e.g. in proton beam therapy where magnets direct protons to strike tumors), in electric motors and generating stations, in older (non-flat screen) TVs and computer monitors, and in other applications. The same process also occurs naturally, notably when the Earth's magnetic field deflects charged particles arriving from the sun to create northern and southern lights. It is possible that "magnetic bottles" will someday enable us to contain the hot electrically charged plasmas which could power hydrogen-fusion power plants.



Maintenance of ION Accelerator  
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The core equation which describes the force acting on an electrically charged particle as it moves in a magnetic field is  $\mathbf{F} = q \mathbf{v} \times \mathbf{B}$ , where  $q$  is the charge,  $\mathbf{v}$  is the particle's vector velocity,  $\mathbf{B}$  is the magnetic field vector and  $\mathbf{F}$  is the resulting vector force. "x" in the equation represents the cross product (or "vector product") of the vectors,  $\mathbf{v}$  and  $\mathbf{B}$ . The equation is inherently 3 dimensional, since it is the component of  $\mathbf{B}$  perpendicular to  $\mathbf{v}$  which produces the force. The resulting force,  $\mathbf{F}$ , is perpendicular to both  $\mathbf{v}$  and  $\mathbf{B}$  in the sense given by the "right hand rule." The 3-dimensional geometry can be confusing, particularly if you are used to thinking of things only on 2-dimensional sheets of paper. This activity will help you sort it out.

**Task:** In this activity you will create a simulation to show how a moving electron, proton, or ion will respond to the magnetic fields you apply. You will then use magnets to steer the "particles" in various ways.

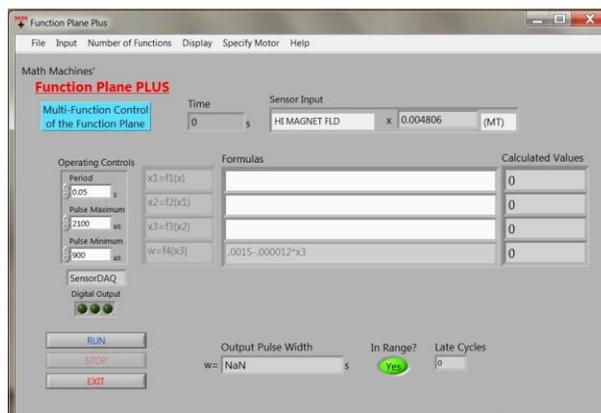
**Additional Materials:** Paper protractor with vector, pointer, magnetic field probe, magnet, masking tape.

## Math Machines Program:

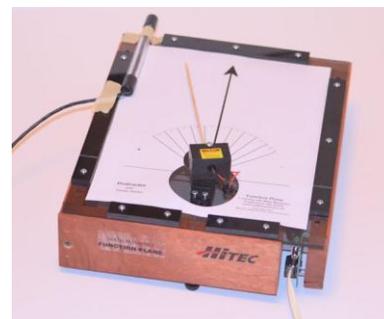
### Function Plane Plus

**Activity Files:** MagneticField01

Plug the Magnetic Field Probe into channel 1 of the SensorDAQ and set the probe on its "High" range.. Run the program FunctionPlanePlus and answer yes in the "Analog Probe in Ch.1" window. Load Activity File MagneticField01.



On the sheet of paper provided, the vector represents the velocity of a particle with electrical charge  $q$  as it moves in a magnetic field. Lay the paper on the Function Plane.



1. Assume first that the charge,  $q$ , is positive as it would be for a proton or an alpha particle. Directly on the paper, draw the direction of  $\mathbf{B}$  which would cause a positively charged particle to deflect to its left. For your drawing, pick one of the symbols below and repeat it several times on the paper.

→	Right
←	Left
↑	Forward (in the same direction as $\mathbf{v}$ )
↓	Back (opposite the direction of $\mathbf{v}$ )
⊙	Out of the paper
X	Into the paper

2. Note that the magnetic field sensor has a white dot<sup>1</sup> to aid in its orientation. The sensors are constructed to measure the strength of a magnetic field,  $\mathbf{B}$ , which leaves the sensor through the white dot. Orient the sensor so that it will measure the magnetic field in the direction you drew on the paper. (The sensor will also measure magnetic fields in the opposite direction and give them a negative value, but it will not respond to fields or components of fields which are tangent to the dot rather than passing through it.) Use masking tape to attach the sensor to the rim of the Function Plane. Bring the south pole of a magnet near the white dot on the sensor and verify that it produces a positive reading for the sensor output. Record below the maximum magnitude for  $\mathbf{B}$  which you observe using your magnet.

**Important note:** Most permanent magnets produce fields that are no more than a few milliteslas strong, properly abbreviated as “mT.” Since magnet fields in the megatesla (MT) range are never encountered—except perhaps in very extreme environments such as near a black hole—some vendors (including Vernier Software and Technology) use the easier-to-print abbreviation “MT” when they should use “mT.”

$$B_{\max} = \text{_____ mT}$$

3. Assume for now that  $q$  is the charge on an alpha particle and that  $\mathbf{v}$  is half the speed of light. Calculate the magnitude of the maximum force which this particle would experience in your maximum magnetic field. Show your work below, including both the equation you use to find  $\mathbf{F}$  and the numeric result for  $F_{\max}$ . The resulting value for  $F$  should be in the standard SI unit of newtons

$$F = \text{_____}$$

$$F_{\max} = \text{_____ N}$$

<sup>1</sup> The white dot is located either on the side of the sensor or on its end, depending on the model.

- Round  $F_{\max}$  up to some convenient value with no more than 2 significant figures, and make this number the maximum scale value for your gauge. Use proportions to create an equation in the form  $a = c F$ , where “a” is the gauge position in degrees and F is the force in newtons. Evaluate the constant, “c,” so the maximum scale value you selected above for force will correspond to an angle of  $+50^\circ$ .

Maximum scale value for force = \_\_\_\_\_

$$a = \text{_____} * F$$

- Add labels to the protractor so it shows Force. Although the calculations are all in newtons, select a more convenient unit for the scale labels, such as mN or  $\mu\text{N}$ .

- In the first function box, enter a function,  $x_1 = f_1(x)$ , which will change the units of the probe reading from mT to the SI base unit of tesla (T). (This is the same as converting millimeters to meters.). Also record the function in the box above exactly as you enter it into the computer.

Magnetic Field (T)  
 Force (N)  
 Angle (degrees)  
 Pulse width (s)

$x_1 = f_1(x) =$	
$x_2 = f_2(x_1) =$	
$x_3 = f_3(x_2) =$	
$x_4 = f_1(x_3) =$	.0015-.000012*x3

- Enter your equation for F as a function of B (from step 3 above) as the second function,  $x_2 = f_2(x_1)$ , where “x1” is the value for B in teslas calculated in the previous function. Also record the function in the box above.
- Enter your scaling equation for a as a function of F (from step 4 above) as the third calculation.  $x_3 = f_3(x_2)$  and also record it above. Taken together, these three functions will convert any value of B produced by the probe into a gauge position (in degrees) which shows the force on an alpha particle moving in this B field at half the speed of light. The fourth function converts this angle into the “pulse width” signal needed by the servo motor.
- What is the domain for this function (the set of possible values for magnetic field, B, in this simulation)?

\_\_\_\_\_

- What is the range for the output of this simulation (the set of possible values of force, F, which can be displayed)?

\_\_\_\_\_

11. Run the simulation and experiment with different ways of holding the magnet to produce substantial positive deflections for the “particle.” Draw sketches below to show three different ways of holding the magnet. Each diagram should clearly show the direction of the magnetic field lines as they leave the magnet and pass through the magnetic field sensor’s white dot.

a.	b.	c.
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12. Find 3 different ways to hold the magnet which cause the particle to deflect to its right, instead of to its left. Sketch each way below, clearly showing the magnetic field lines

a.	b.	c.
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13. Modify your equation for  $F$  as a function of  $B$  to show the force on a proton, instead of an alpha particle. Assume velocity is still half the speed of light and keep the scaling the same. Show your revised function below, and enter it into the control software. Test the simulation with the magnet as before and describe briefly how the force on a proton compares to the force on the alpha particle.

14. The simulation has only considered the force on the moving charged particle, not the particle’s acceleration. If a proton and alpha particle both move at the same speed in the same magnetic field, which will accelerate more? Explain your answer.

