The Power of Logarithms

In 1935, Charles Richter established the "Richter Scale" for measuring earthquakes, defining the magnitude of an earthquake as $M = \log_{10}(d)$, where d is the maximum horizontal movement in micrometers at a distance of 100 km from the epicenter. For example, a movement of 1 mm (1000 micrometers) corresponds to a magnitude 3 earthquake. Modern standards for the design of buildings, dams and other structures now use a different system based on the maximum *acceleration* of the ground, not the maximum movement, but a modified version of the Richter scale is still widely used in reporting the strength of earthquakes.

In addition to the strength of earthquakes, logarithmic scales are widely used to describe a variety of other important—and sometimes damaging—physical events. Examples include the decibel scale for the loudness of sounds, the pH scale for acidity, signal-to-noise ratios in communications, bacterial growth in medicine and virtually anything else which changes over many orders of magnitude. If the ratio between the largest likely value and the smallest interesting value is more than a thousand or so, STEM professions probably use logarithms to describe it.

Magnitude 7.9 Earthquake Damage
Sichuan, China 2008
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Task: In this activity you will design and test a machine which creates scale-model earthquakes of any desired magnitude, M

Additional Materials: ruler or meter stick, thin wooden board to represent the Earth's mantle, small balsa block to represent a portion of the Earth's crust, aluminum knitting needle (US #2, 2.75 mm), paper protractor, and 4 small blocks to represent a multistory building.

Math Machines Program:

Function Plane Plus

Activity Files: Earthquake

Run the program FunctionPlanePlus and answer no in the "Analog Probe in Ch.1" window[1]. Load Activity File Earthquake.


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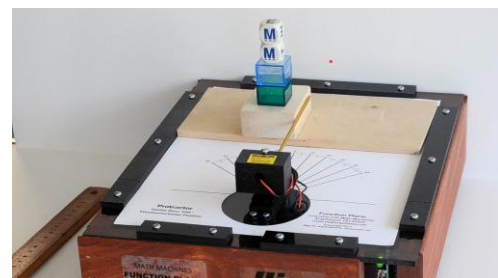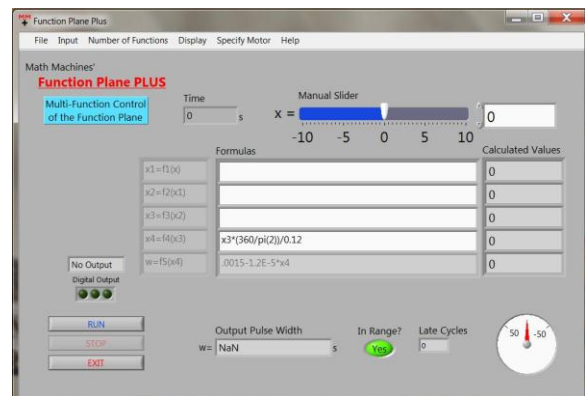
[1] As described here, you will set the magnitude of each earthquake using an on-screen slider. The activity can be modified to instead use a voltage probe and a variety of batteries (from 1.5 to 9 volts) to provide the input.

Several mathematical steps are required to make the Function Plane simulate an earthquake of any magnitude. In the Function Plane Plus software, magnitude M will be set as "x" using a slider at the top of the screen. The end result of the calculations is the pulse width, w, which directs the Function Plane's servo motor to the specific angular position which corresponds to an earthquake of that magnitude.

1. <u>Solve Richter's equation</u>. For his research, Richter defined magnitude, M, as a function of distance, d, or "M = f(d) = $\log_{10}(d)$". For our simulation, however, we need to calculate the distance, d, for any magnitude, M. Solve Richter's equation to show d = f(M). Show your method and your results below.

   d = f(M) = _____

2. <u>Notation</u>. Enter the resulting expression for f(M) into the first formula block in the software, using the notation required by the software. Replace "M" with "x" and replace "d" with "x1". Any exponents must be entered using the symbol "^". For example, "2^3" would be entered as "2^3".

Formulas	
x1=f1(x)	
x2=f2(x1)	
x3=f3(x2)	
x4=f4(x3)	x3*(360/pi(2))/0.12
w=f5(x4)	.0015-1.2E-5*x4
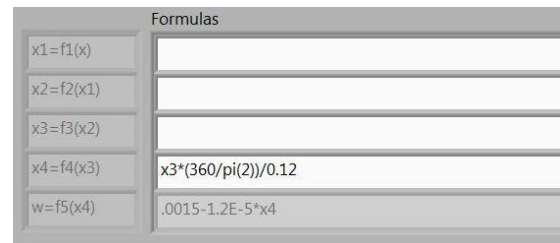
   x1 = f1(x) = _____

3. <u>Unit conversion</u>. Richter's function uses the distance, d, in micrometers, where there are 1 million (1×10^6) micrometers in a meter. Find a function, x2 = f(x1), which will convert the horizontal movement from micrometers into standard units of meters. Enter the result below and enter it into the software's second formula block.

   x2 = f2(x1) = _____

4. <u>Scaling</u>. The result of formula 2 above will be the actual horizontal movement at a distance of 100 km from the epicenter of the earthquake. This value must be reduced to a scale which corresponds to the model we are using. Assume that each story of the scale model building represents a height of 3 m (about 10 feet). Use proportions to reduce the value of x2 by the same ratio that is used in our scale model. Enter the result below and in formula box 3.

   Height of a real 4-story building = _____ meters
   Height of the scale-model of a 4-story building = _____ cm = _____ meters
   Ratio of scale to real = _____
   Movement of the scale model, x3 = f3(x2) = _____

5. <u>Angle in degrees</u>. The distance around a circle is 360° or 2πr. The distance (x4 in degrees) which the motor should turn is proportional to the distance (x3 in meters) which was calculated above, x4/360 = x3 / 2πr. This calculation has already been entered into the software's formula box 4 using the necessary syntax[2], but you may need to modify the value for the radius, r.

   Radius of movement for the scale-model building = _____ cm = _____ m
   (Measure from the pivot point of the motor to the center of the building.)

   x4 = f4(x3) = _____

6. <u>Electronic signal</u>. The Function Plane's servo motor's position is controlled by the width, w, of electronic pulses. The formula to calculate the necessary pulse width, w, for any desired position, x4, has been entered into formula box 5.

   w = f5(x4) = 0.0015 – 1.2E-5*x4

7. <u>Test the Simulator</u>. Create a series of earthquakes and answer each question below.

   a. What magnitudes are almost certain to cause catastrophic damage?

   b. What magnitudes are likely to cause damage but not to destroy the building?

   c. Can there ever be an earthquake with a negative magnitude? If so, how dangerous is an earthquake with M = -7?

8. CHALLENGE: If you can program in LabVIEW, wire the calculations into the sub-vi "w_f(x)" and use it to control the simulation. Print an image of your block diagram and submit it with your report.

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[2] The software requires that "2π" be entered as "pi(2)".